CBCS SCHEME

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17MAT21

Second Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics - II

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Solve : $(4D^4 - 8D^3 - 7D^2 + 11D + 6)\,y = 0$. **(06 Marks)**

b. Solve : $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x}$. **(07 Marks)**

c. Using the method of undetermined coefficients, solve
$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = e^{3x} + \text{Sin}\, x$. **(07 Marks)**

OR

2 a. Solve : $(D^2 + D + 1)y = 1 - n + x^2$. **(06 Marks)**

b. Solve $(D-1)^2 y = e^x + x$. **(07 Marks)**

c. Apply the method of variation of parameters to solve $(D^2 - 6D + 9)\,y = \frac{e^{3x}}{x^2}$. **(07 Marks)**

Module-2

3 a. Solve : $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$. **(07 Marks)**

b. Solve $p(p + y) = x(x + y)$. **(07 Marks)**

c. Obtain the general solution and the singular solution of the following equation as Clairaut's equation : $xp^3 - yp^2 + 1$. **(06 Marks)**

OR

4 a. Solve : $(2x + 3)\, y'' - (2x + 3)\, y' - 12y = 6x$. **(07 Marks)**

b. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form taking the substition as $X = x^2$, $Y = y^2$. **(07 Marks)**

c. Solve : $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. **(06 Marks)**

Module-3

5 a. Form the Partial differential equation by eliminating constants from
$(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$, where α is a known constant. **(06 Marks)**

b. Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ given that $u = 0$ when $t = 0$ and $\frac{\partial u}{\partial t} = 0$ at $x = 0$. Also show that $u \to \sin x$ as $t \to \infty$. **(07 Marks)**

c. Derive one dimensional wave equation in the form
$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **(07 Marks)**

OR

6 a. Obtain the partial differential equation from the following equation by eliminating the arbitrary function $Z = f(x) + e^y g(x)$. **(06 Marks)**

b. Solve the equation $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that $Z = e^y$ and $\frac{\partial z}{\partial x} = 1$, when $x = 0$. **(07 Marks)**

c. Use the method of separation of variables to solve the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

Module-4

7 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a} (x^2 + y^2 + z^2)\, dz\, dy\, dx$. **(06 Marks)**

b. Change the order of integration and hence evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dx\, dy.$$ **(07 Marks)**

c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$. **(07 Marks)**

OR

8 a. Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\, dx\, dy$ by changing to polars. **(06 Marks)**

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$ by changing order of integration. **(07 Marks)**

c. Derive the relation between Beta and Gamma functions as

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$ **(07 Marks)**

Module-5

9 a. Find the Laplace transform of

$$\left[\frac{1-e^{-at}}{t}\right] + t^3 \cosh 4t.$$ **(06 Marks)**

b. Find the Laplace transform of square wave function defined by

$$f(t) = \begin{cases} 1 & \text{if } 0 < t < a \\ -1 & \text{if } a < t < 2a \end{cases}$$ with period 2a. **(07 Marks)**

c. Find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$ using Convolution theorem. **(07 Marks)**

OR

10 a. Express the following function in terms of Unit step function and hence find its Laplace transform

$$f(t) = \begin{cases} t^2 & 0 < t \le 2 \\ 4t & t > 2 \end{cases}.$$ (06 Marks)

b. Find $L^{-1} \log\left[\frac{s^2+1}{s^2+4}\right]$. (07 Marks)

c. Using Laplace transform solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4$, given that $y(0) = 2$, $y'(0) = 3$. (07 Marks)

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